



## Using MSDI and M3Y Core Polarization for the Coulomb Electron Scattering for some Ground state nuclei

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### Abstract

The analysis of Coulomb electron scattering for the ground state-nuclei  $\text{Li}^6$ ,  $\text{C}^{12}$ ,  $\text{O}^{18}$ ,  $\text{F}^{19}$  and  $\text{Ne}^{20}$  are performed with core polarization effects for the Modified Surface Delta Interaction (MSDI) and realistic Michigan three-range Yukawa (M3Y) interaction. The basic calculations included the Coulomb form factor, charge density, and the charge radii of the ground states. The Coulomb form factors for the M3Y interaction gave the best fit with the experimental data.

### Key Words:

*Electron scattering  
 Coulomb form factor  
 Core polarization  
 Ground state*

### Introduction

Electron-nucleus scattering is the deflection of the path of electrons by a target nucleus from their original trajectory. The reason behind this deflection belongs to the negative charge and the low-mass of the electron. Therefore, it can easily be deflected under the effect of the nuclear electromagnetic field when passing near the atomic positive nucleus. This electrostatic (Coulomb) interaction, making electron scattering phenomena. In this process momentum  $q$  and energy  $E$  transfer from the incident electron to the target nucleus, and thus electron is scattered by an angle. According to the experiments, the final state of the nucleus is unknown [1,2].

One of the successful models for describing static properties of nuclei is a shell model within a restricted model space which uses effective charges. Cohen-Kurath [3] model for p-shell nuclei illustrates the low-energy properties very well. For higher energy, Wildenthal universal SD-shell interaction (USD-interaction) is used [4]. The p-shell and SD-shell models which used for testing ground and excited states inadequate to demonstrate electron scattering data, so the effects out of the model space, are necessary to be included in the process of calculation with the effects of core polarization [5].

Comparisons between theoretical calculation and experimental measurement of longitudinal electron scattering form factors have been utilized as a model for transition densities. Early, investigation of theoretical and experimental electrons scattering of  $\text{F}^{19}$  was presented by Brown et al. [6].

Moreover, the nuclear charge density distributions and charge radii are determined from the analysis of elastic electron scattering data [7]. Sharrad *et. al.* have used the ground-state charge density distributions for determining the Coulomb form factor by using the plane-wave Born approximation (PWBA) with the two-body short range correlation [8].

The discussion of the inelastic electron scattering for  $\text{Ne}^{20}$  nucleus has been illustrated by Radhi and Bouchebak [9], and they found that the comparison of predicted C2 and C4 Coulomb form factors have a good agreement with the experimental data. The base of calculations was on the Wildenthal interaction and

Modified surface delta interaction (MSDI) for core polarization effects. In the other hand, Majeed [10] has calculated the longitudinal form factors from  $C^{12}$  nucleus in the framework of the particle-hole shell model and has described all momentum transfer regions of the measured data.

Furthermore, the calculation of the elastic and inelastic electron scattering form factors for  $O^{17,18}$  and  $Ne^{20}$  nuclei with transition probability and charge density distribution by using Skyrme (SKX) and harmonic oscillator (HO) potentials in the Tassie model (core polarization) and model space, have been performed by Jassim and Sahib [11].

Recently, Raheem *et. al.* [12] have been calculated the elastic longitudinal  $C_0$  form factors of some SD-shell nuclei by using effective nucleon-nucleon interaction, which is two-body Michigan three-range Yukawa (M3Y) as residual interactions for the Core Polarization matrix elements.

The aim of this work is to analyze and calculate the ground state charge mean-radii and the Coulomb(longitudinal) electron scattering form factors for the nuclei ( $Li^6$ ,  $C^{12}$ ,  $O^{18}$ ,  $F^{19}$ ,  $Ne^{20}$ ) by considering the role of the core polarization effects for the Modified surface delta interaction (MSDI) and realistic Michigan three-range Yukawa (M3Y) interaction. The harmonic oscillator wave function will be adopted as a single particle wave function. Then we compare the calculated form factors by (MSDI) and (M3Y) interactions with the experimental measurements to show the agreement between them.

FORTTRAN 2008 code used as a computer program for core polarization theoretical calculations. These include modified surface delta interaction (MSDI) and realistic Michigan three-range Yukawa interaction (M3Y) with root mean square charge density and charge radii for the ground state.

## Theory

### 1- Coulomb form factor

The Coulomb form factor of the target nucleus from the initial and final quantum states,  $|J_i\rangle$  and  $|J_f\rangle$  respectively, is the function of the multipolarity J and the transferred momentum ( $q$ ), it is calculated from the relation [13]:

$$F_J(q) = \sqrt{\frac{4\pi}{Z^2(2J_i + 1)}} \langle J_f || O_J(q) || J_i \rangle \quad (1)$$

where  $\langle J_f || O_J(q) || J_i \rangle$  is the reduced many-body matrix element of the electron scattering operator  $O_J(q)$  with the initial and final nuclear spins  $J_i$  and  $J_f$  respectively.

For more precisely calculations, the form factor is corrected for the effects of the center of mass and the nucleon finite-size, or

$$F_J(q) = \sqrt{\frac{4\pi}{Z^2(2J_i + 1)}} \langle J_f || O_J(q) || J_i \rangle e^{\frac{q^2(\beta^2 - 0.43A)}{4A}} \quad (2)$$

Where A is the nuclear mass number, Z is the atomic number, the final term in the above equation is the correction coefficient [14] and  $\beta$  is the size parameter of the harmonic oscillator wave function.

The reduced many-body matrix element of the electron scattering operator, in Eq. (1), is expanded by the single-body matrix elements  $\langle b || O_A^\eta || a \rangle$  and the One-Body-Density Matrix (OBDM) which plays the role of the coefficients of expansion [9] as follow:

$$\langle J_f || O_{J\tau_z} || J_i \rangle = \sum_{a,b} OBDM(a, b, J, \tau_z, i, f) \langle b || O_{J\tau_z} || a \rangle \quad (3)$$

Here, a and b are used to describe single-particle states  $|g\rangle = |n_g l_g j_g m_g\rangle |t_g t_g^z\rangle$ ,  $g \equiv a, b$  for the principal, orbital, total, and projection quantum numbers  $n_g, l_g, j_g$ , and  $m_g$  respectively; while the coefficients  $OBDM(a, b, J, \tau_z, i, f)$  are determined macroscopically by the initial and final nuclear wave functions [15] in neutron-proton formalism. The index  $\tau_z$  (isospin) is to distinguish the nucleons with  $\tau_z = 1, -1$  for protons and neutrons respectively. The single-particle matrix element is determined from [16]:

$$\langle b || O_{J\tau_z} || a \rangle = \langle j_b || Y_{J\tau_z} || j_a \rangle \langle n_b, l_b | j_J(qr) | n_a l_a \rangle, \quad (4)$$

where  $\langle n_b, l_b | j_J(qr) | n_a l_a \rangle$  is the radial matrix element of the spherical Bessel functions  $j_J(qr)$  which is calculated in Eq. (23) of our published paper [17]; and  $\langle j_b || Y_{J\tau_z} || j_a \rangle$  represents the reduced matrix element of the spherical harmonics  $Y_{J\tau_z}$  ( $\tau_z = 1, -1$  for protons and neutrons respectively).

For ground states, there are no transitions to excited states, it means  $J_i = J_f$  and the multipolarity  $J = 0$ .

In this case, the reduced matrix element is simplified to  $\langle j_b || Y_\lambda(\Omega_r) || j_a \rangle = \delta_{j_a j_b} \sqrt{(2j_a + 1)/4\pi}$  [16], then Eq. (3) is re-expressed as:

$$\langle J_i || O_{J\tau_z} || J_i \rangle = \sum_a OBDM(a, a, 0, \tau_z, i, f) \langle a || O_{J\tau_z} || a \rangle \quad (5)$$

The reduced matrix element in Eq. (1) comes from the sum of two effects, model space (MS) and core polarization (CP). For the ground state  $J = 0$  and  $J_i = J_f$ , it is given by:

$$\langle J_i || O_{J\tau_z}(q) || J_i \rangle = \langle J_f || O_{J\tau_z}(q) || J_i \rangle_{MS} + \langle J_f || \delta O_{J\tau_z}(q) || J_i \rangle_{CP} \quad (6)$$

In Eq. (6), the first term (MS) is directly calculated from Eq. (3), and similarly, the second term (CP) can be obtained with Eq. (7). The only difference between the reduced-matrix elements (MS) and (CP) belongs to the single-particle matrix elements [9]

$$\langle J_i || \delta O_{J\tau_z}(q) || J_i \rangle_{CP} = \sum_a OBDM(a, a, 0, \tau_z, i, f) \langle a || \delta O_{J\tau_z} || a \rangle \quad (7)$$

In the above equation the single-particle matrix element  $\langle a || \delta O_{J\tau_z} || a \rangle$  is obtained from particle-hole excitation with first-order perturbation including the two-body interaction (V) for the Modified Surface Delta Interaction (MSDI) and (M3Y) [15]

$$\begin{aligned} \langle b || \delta O_\Lambda || a \rangle_{CP} &= \left\langle b \left| \left| V \frac{Q}{E_a - H_o} O_\Lambda \right| \right| a \right\rangle \\ &+ \left\langle b \left| \left| O_\Lambda \frac{Q}{E_b - H_o} V \right| \right| a \right\rangle \end{aligned} \quad (8)$$

With

$$\begin{aligned} \left\langle \beta \left| \left| V \frac{Q}{E_b - H_o} O_\Lambda \right| \right| \alpha \right\rangle &= \sum_{\Gamma, \alpha_1, \alpha_2} \frac{\langle \beta \alpha_2 | V | \alpha \alpha_1 \rangle_\Gamma \langle \alpha_1 || O_\Lambda || \alpha_2 \rangle}{e_\alpha - e_\beta + e_{\alpha_1} - e_{\alpha_2}} (-1)^{\alpha + \alpha_2 + \Gamma} (2\Gamma + 1) \\ &\times \sqrt{(1 + \delta_{bp})(1 + \delta_{ah})} \begin{Bmatrix} \beta & \alpha & \Lambda \\ \alpha_1 & \alpha_2 & \Gamma \end{Bmatrix} \\ &+ \text{ terms with } \alpha_1 \text{ and } \alpha_2 \text{ exchanged with an over all minus sign} \end{aligned} \quad (9)$$

Here,  $H_0$  is the unperturbed Hamiltonian and the operator  $Q$  projects the space outside the model space. And  $\left\{ \begin{matrix} \beta & \alpha & \Lambda \\ \alpha_1 & \alpha_2 & \Gamma \end{matrix} \right\}$  is the six-j symbol and the indices  $\alpha_1$  and  $\alpha_2$  run over particle and hole states respectively; we can collect them in  $e_\gamma$  which is the single-particle energy for quantum state  $\gamma$  such that  $\gamma = \alpha, \beta, \alpha_1, \alpha_2$ . All the matrix elements at the left side in Eq. (8) are obtained in isoscalar ( $T = 0$ ) and isovector ( $T = 1$ ) formalism with  $\Lambda = J, T$  and  $\Gamma = J', T'$ .

## 2- Charge density and charge radii

The operator of transition charge density of a nucleus is expressed in terms of the sum of all point charges for the protons [13]:

$$\hat{\rho}_{JM}(\vec{r}) = \sum_{k=1}^Z \frac{\delta(\vec{r} - \vec{r}_k)}{r_k} Y_{JM}(\Omega_k) \quad (10)$$

where  $J$  is the multipolarity of the operator,  $M$  represents its projection quantum number takes  $2J + 1$  values,  $-J \leq M \leq J$ ,  $Y_{JM}(\Omega_k)$  is the spherical harmonic and  $\delta(\vec{r} - \vec{r}_k)$  Dirac delta functions.

The reduced matrix element of the operator  $\hat{\rho}_{JM}(\vec{r})$  is obtained when it undergoes the transition from initial nuclear spin  $J_i$  to the final nuclear spin  $J_f$  and satisfying the inequality  $J_i \leq J \leq J_f$ , see Eq. (3) where  $O_{J\tau_z} \equiv \hat{\rho}_J(\vec{r})$ , then it is given by:

$$\langle J_f || \hat{\rho}_J(\vec{r}) || J_i \rangle = \sum_{a,b} OBDM(a, b, J, \tau_z, i, f) \langle b || \hat{\rho}_J(\vec{r}) || a \rangle \quad (11)$$

and the nuclear charge density  $\rho_J^p(r)$  is obtained from the matrix element [16]:

$$\rho_J^p(r) = \frac{1}{\sqrt{4\pi(2J_i + 1)}} \langle J_f || \hat{\rho}_J(\vec{r}) || J_i \rangle = \frac{1}{\sqrt{4\pi(2J_i + 1)}} \sum_{a,b} OBDM(a, b, J, \tau_z, i, f) \langle b || \hat{\rho}_J(\vec{r}) || a \rangle \quad (12)$$

The single-particle matrix element in Eq. (3) can be expressed by the harmonic oscillator radial wave functions  $\mathcal{R}_{n_a l_a}(r)$   $\mathcal{R}_{n_b l_b}(r)$  and the reduced matrix element of the spherical harmonic functions  $\langle l_b j_b || Y_J(\Omega_r) || l_a j_a \rangle$

$$\langle b || \hat{\rho}_J(\vec{r}) || a \rangle = \mathcal{R}_{n_a l_a}(r) \mathcal{R}_{n_b l_b}(r) Y_J(\Omega_r) \quad (13)$$

Substituting Eq. (13) in Eq. (12), the nuclear charge density gives

$$\rho_J^p(r) = \frac{1}{\sqrt{4\pi(2J_i + 1)}} \sum_{a,b} OBDM(a, b, J, \tau_z, i, f) \mathcal{R}_{n_a l_a}(r) \mathcal{R}_{n_b l_b}(r) \langle j_b || Y_J(\Omega_r) || j_a \rangle \quad (14)$$

For the ground state nucleus, ( $\tau_z = 1, -1$ ), it leads to  $J = 0$  and  $\langle j_b || Y_J(\Omega_r) || j_a \rangle = \delta_{j_a j_b} \sqrt{(2j_a + 1)/4\pi}$ . After using the Delta-Kronecker in Eq. (14) and putting  $\tau_z = 1$  for protons, the equation is re-expressed as

$$\rho_0^p(r) = \frac{1}{\sqrt{4\pi(2J_i + 1)}} \sum_a OBDM(a, a, 0, \tau_z, i, f) \sqrt{(2j_a + 1)} |R_{n_a l_a}(r)|^2 \quad (15)$$

the index  $a \equiv n_a l_a j_a$  is used for all closed shells for the ground state.

Charge radii and charge distribution of ground state nuclei are considered as two good measurable quantities experimentally, in the same time, they can be calculated theoretically. The root-mean-square for the nucleus is obtained from the integration of charge density in Eq. (15) [18,19].

$$\langle r^2 \rangle_{ch} = \int \rho_0^p(\vec{r}) r^2 d\vec{r} \quad (16)$$

Under effect of the folded charge of the point proton, the charge density in Eq. (8) must be corrected by the folding factor [18]:

$$\rho_{fo}(\vec{r} - \vec{r}') = \frac{1}{\sqrt{\pi^3} \alpha^6} e^{-\frac{(\vec{r}-\vec{r}')^2}{\alpha^2}} \quad , \alpha = 0.6532 \text{ fm}^2 \quad (17)$$

Now, for the normalized charge density with the atomic number of the target nucleus Z, the root-mean-square in Eq. (16) gives:

$$\langle r^2 \rangle_{ch} = \frac{1}{Z} \int \rho_0^p(\vec{r}) \rho_{fo}(\vec{r} - \vec{r}') r^2 d\vec{r} \quad (18)$$

### Result and Discussion

The present study includes the core polarization effects to explain the experimental data of electron scattering through the Born approximation that is the mathematical tool with the first-order perturbation in the calculations of core polarization.

The ground states of nuclei are defined by the initial spin-parity-isospin notation  $J_i^{\pi_i} T_i$  and there is no transition from the ground state to the excited states resulting to give zero spin-multipolarity ( $J = 0$ ) from the triangular inequality formula  $J_i \leq J \leq J_i$ .

In the calculation of nuclear charge density Eq. (15), the harmonic oscillator wave functions  $\mathcal{R}_{n_a l_a}(r)$  are used with size parameters  $b = 1.88 f$  for  $\text{Li}^6$  [20],  $b = 1.692 f$  for  $\text{C}^{12}$  [21],  $b = 1.841 f$  for  $\text{O}^{18}$  [22],  $b = 1.833 f$  for  $\text{F}^{19}$  and  $b = 1.869 f$  for  $\text{Ne}^{20}$  [23].

The strength parameters of the MSDI interaction used in the calculations of the core polarization effects are  $A_T$ , B and C. Where T is the isospin (1,0). They are taken as  $A_0 = A_1 = B = 25/A$  and  $C = 0$  [15], where A is the mass number of the nucleus, B and C are the correction parameters. And the parameters of M3Y which known as three-range potential contain spin-orbit, central and tensor interactions are from Bertsch *et. al.* [24].

The one-body density matrixes (OBDM) are obtained from the occupation numbers of closed shell orbits and the values of (OBDM) for the closed-shell protons are listed in Table (1).

Table-1: The calculated one body density matrix for the closed-shell protons.

$a (n_a l_a j_a)$	$\text{Li}^6$	$\text{C}^{12}$	$\text{O}^{18}$	$\text{F}^{19}$	$\text{Ne}^{20}$
1s <sub>1/2</sub>	2.4495	1.4142	1.4142	2.00	1.4142
1p <sub>3/2</sub>	....	....	2.00	2.8284	2.00
1p <sub>1/2</sub>	....	....	1.4142	2.00	1.4142

The root-mean-squares for charge density with-and-without folding effect is calculated in Eq. (18) and Eq. (16) respectively, the calculated charge radii are compared with the experimental values and listed in Table (2).

Table-2: The calculated and measured charge radii for the ground state nuclei.

Nuclei	Charge radii in Fermi		
	Without folding	With folding	Exp. [25]
${}^3\text{Li}^6$	2.657	2.534	2.589
${}^6\text{C}^{12}$	2.594	2.467	2.470
${}^8\text{O}^{18}$	2.875	2.761	2.772
${}^9\text{F}^{19}$	2.939	2.828	2.897
${}^{10}\text{Ne}^{20}$	3.052	2.945	3.005

Both nuclei  $\text{Li}^6$ ,  $\text{C}^{12}$  have two nucleons and eight nucleons outside the core  $\text{He}^4$  respectively. The two nucleons of  $\text{Li}^6$  and eight nucleons of  $\text{C}^{12}$  are distributed over p-shell model space. The ground state of  $\text{Li}^6$  is ( $J^{\pi T} = 1^+0$ ) and for  $\text{C}^{12}$  is ( $J^{\pi T} = 0^+0$ ). And the nucleus  $\text{O}^{18}$  consist of an inert  $\text{O}^{16}$  core plus two nucleons,

$F^{19}$  consist of inert  $O^{16}$  core plus three nucleons, and  $Ne^{20}$  has four nucleons outside the core  $O^{16}$ . All the outside electrons of these nuclei are distributed over SD-shell model space. The ground state of them are ( $J^{\pi}T = 0^{+}1$ ), ( $J^{\pi}T = \frac{1}{2}^{+}\frac{1}{2}$ ) and ( $J^{\pi}T = 0^{+}0$ ) respectively.

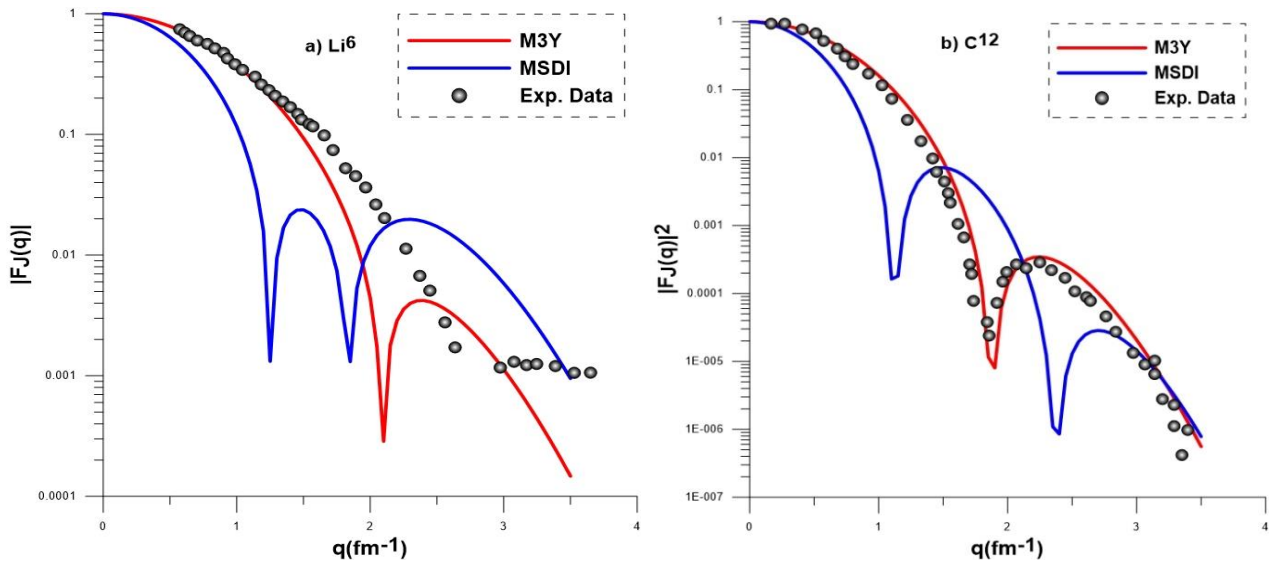
For the ground states, the Coulomb form factors with a transition ( $C_0$ ) are calculated in Eq. (2) for the range of transferred momentum  $0 \leq q \leq 4 \text{ fm}^{-1}$ . For all nuclei, the squared form factors are plotted with the transferred momentum in Fermi unit, except for the  $Li^6$  nucleus where the plot is between the first order of form factor and transferred momentum ( $q$ ). And compared with the experimental values as shown in Figure (1). Otherwise, the curves do not appropriate with each other. The blue lines represent the form factors with MSDI core polarization; while the red lines represent the form factors with M3Y core polarization and the spherical symbols represent the experimental data.

In Figure (1-a), the  $C_0$  form factor of  $Li^6$  nucleus displays, including MSDI and M3Y core polarization. We observe that the calculation with M3Y gives the best fit with the experimental data in the interval  $q \leq 1.4 \text{ fm}^{-1}$ , but the calculated value of MSDI underestimated with the experimental data until the region  $q \leq 2.3 \text{ fm}^{-1}$  where they crossed each other only at one point.

Figure (1-b) displays the  $C_0$  transition of form factor for  $C^{12}$  nucleus with MSDI and M3Y core polarization. The M3Y calculation gives completely a good description of the experimental data, but the MSDI calculation is mainly different from the data.

The plot of the ground state form factor of  $O^{18}$  is shown in Figure (1-c) for the residual interactions MSDI and M3Y. The experimental data cover the low region of the transferred momentum and the data completely situated between the MSDI and M3Y curves.

The Figures (1-d) and (1-e) show the calculated form factors with both (MSDI), (M3Y) interactions and experimentally measured form factor in three plots for  $F^{19}$  and  $Ne^{20}$  nuclei respectively. We conclude that for  $F^{19}$  nucleus, the result of (M3Y) interaction has a good description for the experimental data, especially between the ( $0.5 \leq q \leq 2.5 \text{ fm}^{-1}$ ) values. Whereas in the plots of  $Ne^{20}$  nucleus, core polarization with (M3Y) interaction has given a good fit for the experimental data for some specific regions of the transferred momentum.



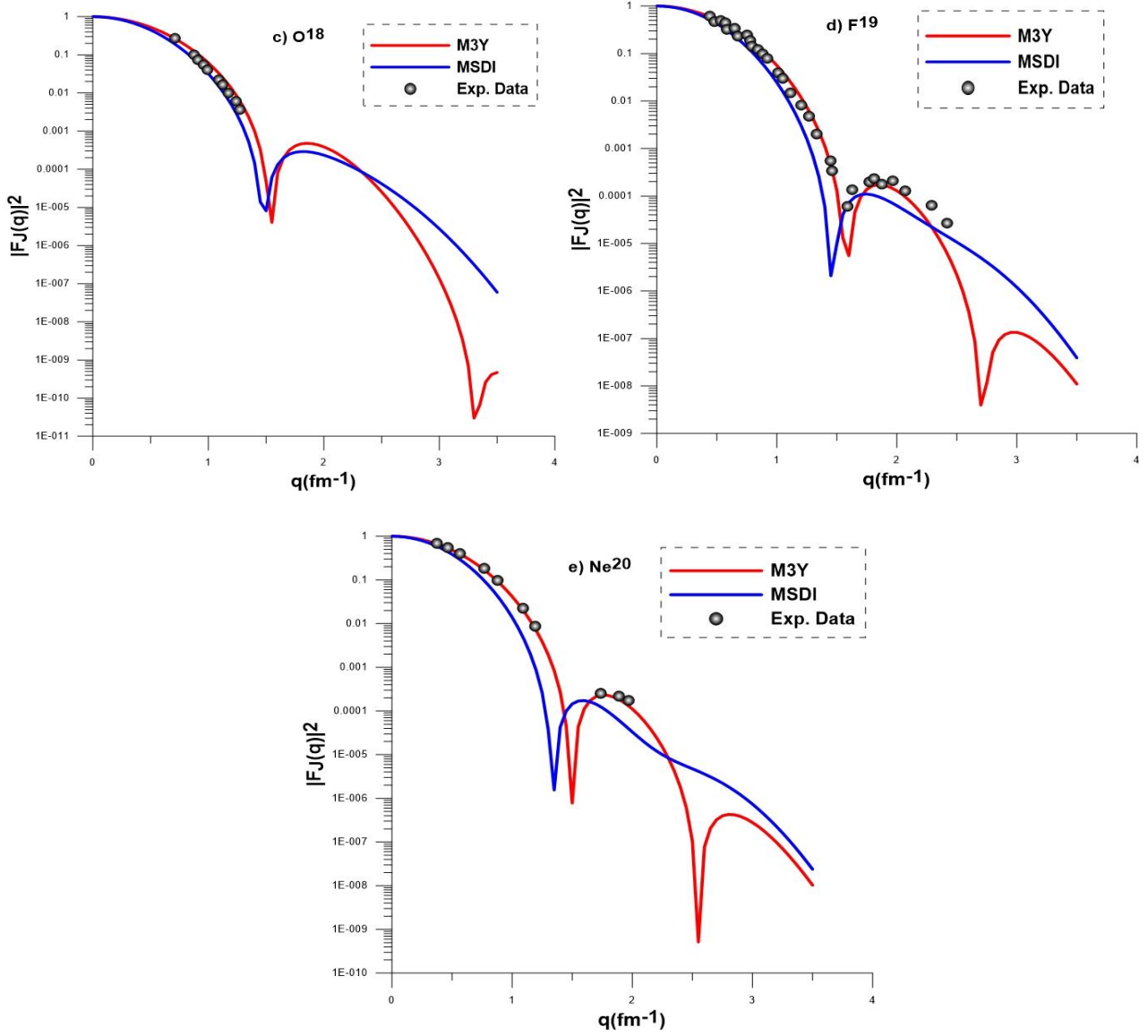


Figure-1: Plot of the elastic Coulomb form factors ( $C_0$ ) using MSDI and M3Y core polarizations with the experimental data for nuclei  $\text{Li}^6$  [26],  $\text{C}^{12}$  [27],  $\text{O}^{18}$  [22],  $\text{F}^{19}$  [28], and  $\text{Ne}^{20}$  [29].

## Conclusion

The p-shell and SD-shell model spaces and harmonic oscillator wave functions are adopted to compute the elastic  $C_0$  form factors for the nuclei  $\text{Li}^6$ ,  $\text{C}^{12}$ ,  $\text{O}^{18}$ ,  $\text{F}^{19}$ , and  $\text{Ne}^{20}$ . Two residual interactions are utilized as residual interactions in this work, for the core-polarization terms, which are the MSDI and M3Y effective interactions. From the results of the present study, one concludes that the theoretical elastic Coulomb form factors with M3Y interaction can describe the experimental data better than MSDI interaction. Because M3Y interaction is more realistic nucleon-nucleon interaction that adopted for the CP calculation. That is mean the interaction depends on all nucleons, but MSDI interaction is dealing with the surface nucleons only.

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